Exercise 2.1

Question 1:

Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.

(i) $4x^2 - 3x + 7$

(ii) $y^2 + \sqrt{2}$

(iii) $3\sqrt{t} + t\sqrt{2}$

(iv) $y + \frac{2}{y}$

(v) $y + 2y^{-1}$

Solution 1:

i) $4x^2 - 3x + 7$

One variable is involved in given polynomial which is 'x'

Therefore, it is a polynomial in one variable 'x'.

(ii) $y^2 + \sqrt{2}$

One variable is involved in given polynomial which is 'y'

Therefore, it is a polynomial in one variable 'y'.

(iii) $3\sqrt{t} + t\sqrt{2}$

No. It can be observed that the exponent of variable t in term $3\sqrt{t}$ is $\frac{1}{2}$, which is nota whole number. Therefore, this expression is not a polynomial.

(iv) $y + \frac{2}{y}$

$= y + 2y^{-1}$

The power of variable 'y' is -1 which is not a whole number.

Therefore, it is not a polynomial in one variable

No. It can be observed that the exponent of variable y in term $\frac{2}{y}$ is -1, which is not a whole number. Therefore, this expression is not a polynomial.

(v) $x^{10} + y^3 + t^{50}$

In the given expression there are 3 variables which are 'x, y, t' involved.

Therefore, it is not a polynomial in one variable.

Question 2:

Write the coefficients of x^2 in each of the following:

(i) $2+x^2+x$

(ii) $2-x^2+x^3$

(iii) $\frac{\pi}{2}x^2+x$

(iv) $\sqrt{2}x-1$

Solution 2:

(i) $2+x^2+x^3$

$=2+1(x^2)+x$

The coefficient of x^2 is 1.

(ii) $2-x^2+x^3$

$=2-1(x^2)+x$

The coefficient of x^2 is -1.

(iii) $\frac{\pi}{2}x^2+x$

The coefficient x^2 of is $\frac{\pi}{2}$.

(iv) $\sqrt{2}x-1=0x^2+\sqrt{2}x-1$

The coefficient of x^2 is 0.

Question 3:

Give one example each of a binomial of degree 35, and of a monomial of degree 100.

Solution 3 :

Binomial of degree 35 means a polynomial is having

1. Two terms
2. Highest degree is 35

Example: $x^{35}+x^{34}$

Monomial of degree 100 means a polynomial is having

1. One term
2. Highest degree is 100

Example : x^{100}.

Question 4:

Write the degree of each of the following polynomials:

(i) $5x^3+4x^2+7x$

(ii) $4-y^2$

(iii) $5t-\sqrt{7}$

(iv) 3

Solution 4:

Degree of a polynomial is the highest power of the variable in the polynomial.

(i) $5x^3+4x^2+7x$

Highest power of variable 'x' is 3. Therefore, the degree of this polynomial is 3

(ii) $4-y^2$

Highest power of variable 'y' is 2. Therefore, the degree of this polynomial is 2.

(iii) $5t-\sqrt{7}$

Highest power of variable 't' is 1. Therefore, the degree of this polynomial is 1.

(iv) 3

This is a constant polynomial. Degree of a constant polynomial is always 0.

Question 5: Classify the following as linear, quadratic and cubic polynomial:

(i) x^2+x

(ii) $x-x^3$

(iii) $y+y^2+4$

(iv) $1+x$
(v) 3t
(vi) r^2
(vii) $7x^2 \quad 7x^3$

Solution 5:
Linear polynomial – whose variable power is '1'

Quadratic polynomial - whose variable highest power is '2'
Cubic polynomial- whose variable highest power is '3'

(i) x^2+x is a quadratic polynomial as its highest degree is 2.
(ii) $x-x^3$ is a cubic polynomial as its highest degree is 3.
(iii) $y+y^2+4$is a quadratic polynomial as its highest degree is 2.
(iv) 1 + x is a linear polynomial as its degree is 1.
(v) 3t is a linear polynomial as its degree is 1.
(vi) r^2 is a quadratic polynomial as its degree is 2.
(vii) $7x^2 \quad 7x^3$ is a cubic polynomial as highest its degree is 3.

Exercise 2.2

Question 1:
Find the value of the polynomial at $5x-4x^2+3$ at
(i) x = 0
(ii) x = −1
(iii) x = 2

Solution 1:
(i) $p(x)=5x-4x^2+3$
$p(0)=5(0)-4(0)^2+3 = 3$

(ii) $p(x)=5x-4x^2+3$

$p(-1)=5(-1)-4(-1)^2+3$
$= -5-4(1)+3=-6$

(iii) $p(x)=5x-4x^2+3$
$p(2)=5(2)-4(2)^2+3 = 10-16+3=-3$

Question 2:

Find p(0), p(1) and p(2) for each of the following polynomials:

(i) $p(y) = y^2 - y + 1$

(ii) $p(t) = 2 + t + 2t^2 - t3$

(iii) $p(x) = x^3$

(iv) $p(x) = (x - 1)(x + 1)$

Solution 2:

(i) $p(y) = y^2 - y + 1$

- $p(0) = (0)^2 - (0) + 1 = 1$
- $p(1) = (1)^2 - (1) + 1 = 1 - 1 + 1 = 1$
- $p(2) = (2)^2 - (2) + 1 = 4 - 2 + 1 = 3$

(ii) $p(t) = 2 + t + 2t^2 - t^3$

- $p(0) = 2 + 0 + 2(0)^2 - (0)^3 = 2$
- $p(1) = 2 + (1) + 2(1)^2 - (1)^3 = 2 + 1 + 2 - 1 = 4$
- $p(2) = 2 + 2 + 2(2)^2 - (2)^3$

$= 2 + 2 + 8 - 8 = 4$

(iii) $p(x) = x^3$

- $p(0) = (0)^3 = 0$
- $p(1) = (1)^3 = 1$
- $p(2) = (2)^3 = 8$

(v) $p(x) = (x - 1)(x + 1)$

- $p(0) = (0 - 1)(0 + 1) = (-1)(1) = -1$
- $p(1) = (1 - 1)(1 + 1) = 0(2) = 0$
- $p(2) = (2 - 1)(2 + 1) = 1(3) = 3$

Question 3:

Verify whether the following are zeroes of the polynomial, indicated against them.

(i) $p(x) = 3x + 1, x = -\frac{1}{3}$

(ii) $p(x) = 5x - \pi, x = \frac{4}{5}$

(iii) $p(x) = x^2 - 1, x = 1, -1$

(iv) $p(x) = (x+1)(x-2), x = -1, 2$

(v) $p(x) = x^2, x = 0$

(vi) $p(x) = lm + m, x = -\frac{m}{l}$

(vii) $p(x) = 3x^2 - 1, x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

(viii) $p(x) = 2x + 1, x = \frac{1}{2}$

Solution 3:

(i) If $x = -\frac{1}{3}$ is a zero of given polynomial p(x) = 3x + 1, then $p\left(-\frac{1}{3}\right)$ should be 0.

Here, $p\left(-\frac{1}{3}\right) = 3\left(-\frac{1}{3}\right) + 1 = -1 + 1 = 0$

Therefore, is a zero of the given polynomial.

(ii) If $x = \frac{4}{5}$ is a zero of polynomial p(x) = 5x − π, then $p\left(\frac{4}{5}\right)$ should be 0.

Here, $p\left(\frac{4}{5}\right) = 5\left(\frac{4}{5}\right) - \pi = 4 - \pi$

As $p\left(\frac{4}{5}\right) \neq 0$

Therefore, $x = \frac{4}{5}$ is not a zero of the given polynomial.

(iii) If $x = 1$ and $x = -1$ are zeroes of polynomial $p(x) = x^2 - 1$, then $p(1)$ and $p(-1)$should be 0.

Here, $p(1) = (1)^2 - 1 = 0$, and

$p(-1) = (-1)^2 - 1 = 0$

Hence, $x = 1$ and -1 are zeroes of the given polynomial.

(iv) If $x = -1$ and $x = 2$ are zeroes of polynomial $p(x) = (x + 1)(x - 2)$, then $p(-1)$ and $p(2)$should be 0.

Here, $p(-1) = (-1 + 1)(-1 - 2) = 0(-3) = 0$, and

$p(2) = (2 + 1)(2 - 2) = 3(0) = 0$

Therefore, $x = -1$ and $x = 2$ are zeroes of the given polynomial.

(v) If $x = 0$ is a zero of polynomial $p(x) = x^2$, then $p(0)$ should be zero.

Here, $p(0) = (0)^2 = 0$

Hence, $x = 0$ is a zero of the given polynomial.

(vi) If $p\left(\frac{-m}{l}\right)$ is a zero of polynomial $p(x) = lx + m$, then $p\left(\frac{-m}{l}\right)$ should be 0.

Here, $p\left(\frac{-m}{l}\right) = l\left(\frac{-m}{l}\right) + m = -m + m = 0$

Therefore, $x = \frac{-m}{l}$ is a zero of the given polynomial.

(vii) If $x = \frac{-1}{\sqrt{3}}$ and $x = \frac{2}{\sqrt{3}}$ are zeroes of polynomial $p(x) = 3x^2 - 1$, then

$p\left(\frac{-1}{\sqrt{3}}\right)$ and $p\left(\frac{2}{\sqrt{3}}\right)$ should be 0.

Here, $p\left(\frac{-1}{\sqrt{3}}\right)=3\left(\frac{-1}{\sqrt{3}}\right)^2-1=3\left(\frac{1}{3}\right)-1=1-1=0$, and

$$p\left(\frac{2}{\sqrt{3}}\right)=3\left(\frac{2}{\sqrt{3}}\right)^2-1=3\left(\frac{4}{3}\right)-1=4-1=3$$

Hence, $x=\frac{-1}{\sqrt{3}}$ is a zero of the given polynomial.

However, $x=\frac{2}{\sqrt{3}}$ is not a zero of the given polynomial.

(viii) If $x=\frac{1}{2}$ is a zero of polynomial $p(x) = 2x + 1$, then $p\left(\frac{1}{2}\right)$ should be 0.

Here, $p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right) + 1 = 1 + 1 = 2$

As $p\left(\frac{1}{2}\right)\neq 0$,

Therefore, $x=\frac{1}{2}$ is not a zero of the given polynomial.

Question 4:

Find the zero of the polynomial in each of the following cases:

(i) $p(x) = x + 5$

(ii) $p(x) = x - 5$

(iii) $p(x) = 2x + 5$

(iv) $p(x) = 3x - 2$

(v) $p(x) = 3x$

(vi) $p(x) = ax$, $a \neq 0$

(vii) $p(x) = cx + d$, $c \neq 0$, c, d are real numbers.

Solution 4:

Zero of a polynomial is that value of the variable at which the value of thepolynomial is obtained as 0.

(i) $p(x) = x + 5$
Let $p(x) = 0$
$x + 5 = 0$
$x = -5$
Therefore, for $x = -5$, the value of the polynomial is 0 and hence, $x = -5$ is a zero of the given polynomial.

(ii) $p(x) = x - 5$
Let $p(x) = 0$
$x - 5 = 0$
$x = 5$
Therefore, for $x = 5$, the value of the polynomial is 0 and hence, $x = 5$ is a zero of the given polynomial.

(iii) $p(x) = 2x + 5$
Let $p(x) = 0$
$2x + 5 = 0$
$2x = -5$
$x = -\frac{5}{2}$

Therefore, for $x = -\frac{5}{2}$, the value of the polynomial is 0 and hence, $x = -\frac{5}{2}$ is a zero of the given polynomial.

(iv) $p(x) = 3x - 2$

$p(x) = 0$
$3x - 2 = 0$
Therefore, for $x = \frac{2}{3}$, the value of the polynomial is 0 and hence, $x = \frac{2}{3}$ is a zero of the given polynomial.

(v) p(x) = 3x
Let p(x) = 0
3x = 0
x = 0
Therefore, for x = 0, the value of the polynomial is 0 and hence, x = 0 is a zero of the given polynomial.

(vi) p(x) = ax
Let p(x) = 0
ax = 0
x = 0
Therefore, for x = 0, the value of the polynomial is 0 and hence, x = 0 is a zero of the given polynomial.

(vii) p(x) = cx + d
Let p(x) = 0
cx + d = 0
$x = \frac{-d}{c}$
Therefore, for $x = \frac{-d}{c}$, the value of the polynomial is 0 and hence, $x = \frac{-d}{c}$ is a zero of the given polynomial.

Exercise 2.3

Question 1:

Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by

(i) $x + 1$

(ii) $x-\frac{1}{2}$

(iii) x

(iv) $x + \pi$

(v) $5 + 2x$

Solution 1:

(i) $x^3 + 3x^2 + 3x + 1 \div x + 1$

By long division, we get

$$
\begin{array}{r|l}
 & x^2+2x+1 \\
\hline
x+1 & \cancel{x^3}+3x^2+3x+1 \\
 & \cancel{x^3}+x^2 \\
 & -\quad - \\
\hline
 & \quad 2\cancel{x^2}+3x+1 \\
 & \quad 2\cancel{x^2}+2x \\
 & \quad -\quad - \\
\hline
 & \qquad \cancel{x}+1 \\
 & \qquad \cancel{x}+1 \\
 & \qquad -\ - \\
\hline
 & \qquad 0
\end{array}
$$

Therefore, the remainder is 0.

(ii) $x^3 + 3x^2 + 3x + 1 \div x-\frac{1}{2}$

By long division,

$$
\begin{array}{r|l}
\multicolumn{2}{r}{x^2+\frac{7}{2}x+\frac{19}{4}} \\
\cline{2-2}
x-\frac{1}{2} & x^3+3x^2+3x+1 \\
 & x^3-\frac{x^2}{2} \\
 & -\quad + \\
\cline{2-2}
 & \frac{7}{2}x^2+3x+1 \\
 & \frac{7}{2}x^2-\frac{7}{4}x \\
 & -\quad + \\
\cline{2-2}
 & \frac{19}{4}x+1 \\
 & \frac{19}{4}x-\frac{19}{8} \\
 & -\quad + \\
\cline{2-2}
 & \frac{27}{8} \\
\cline{2-2}
\end{array}
$$

Therefore, the remainder is $\frac{27}{8}$.

(iii) $x^3+3x^2+3x+1 \div x$

By long division,

$$
\begin{array}{r|l}
\multicolumn{2}{r}{x^2+3x+3} \\
\cline{2-2}
x & x^3+3x^2+3x+1 \\
 & x^3 \\
 & - \\
\cline{2-2}
 & 3x^2+3x+1 \\
 & 3x^2 \\
 & - \\
\cline{2-2}
 & 3x+1 \\
 & 3x \\
 & - \\
\cline{2-2}
 & 1 \\
\cline{2-2}
\end{array}
$$

Therefore, the remainder is 1.

(iv) $x^3 + 3x^2 + 3x + 1 \div x + \pi$
By long division, we get

$$
\begin{array}{r|l}
 & x^2+(3-\pi)x+(3-3\pi+\pi^2) \\
\hline
x+\pi & x^3+3x^2+3x+1 \\
 & x^3+\pi x^2 \\
 & - \quad - \\
\hline
 & (3-\pi)x^2+3x+1 \\
 & (3-\pi)x^2+(3-\pi)\pi x \\
 & - \qquad - \\
\hline
 & [3-3\pi+\pi^2]x+1 \\
 & [3-3\pi+\pi^2]x+(3-3\pi+\pi^2)\pi \\
 & - \qquad - \\
\hline
 & [1-3\pi+3\pi^2-\pi^3] \\
\hline
\end{array}
$$

Therefore, the remainder is $-\pi^3+3\pi^2-3\pi+1$.

(v) $5 + 2x$
By long division, we get

$$
\begin{array}{r|l}
 & \frac{x^2}{2}+\frac{x}{4}+\frac{7}{8} \\
\hline
2x+5 & x^3+3x^2+3x+1 \\
 & x^3+\frac{5}{2}x^2 \\
 & -\quad - \\
\hline
 & \frac{x^2}{2}+3x+1 \\
 & \frac{x^2}{2}+\frac{5x}{4} \\
 & -\quad - \\
\hline
 & \frac{7x}{4}+1 \\
 & \frac{7}{4}x+\frac{35}{8} \\
 & -\quad - \\
\hline
 & -\frac{27}{8} \\
\hline
\end{array}
$$

Therefore, the remainder is $-\frac{27}{8}$.

Question 2:

Find the remainder when $x^3 - ax^2 + 6x - a$ is divided by $x - a$.

Solution 2:

$x^3 - ax^2 + 6x - a \div x - a$

By long division,

$$
\begin{array}{r|l}
 & x^2+6 \\
\hline
x-a & x^3-ax^2+6x-a \\
 & x^3-ax^2 \\
 & -\quad + \\
\hline
 & 6x-a \\
 & 6x-6a \\
 & -\quad + \\
\hline
 & 5a \\
\hline
\end{array}
$$

Therefore, when $x^3 - ax^2 + 6x - a$ is divided by $x - a$, the remainder obtained is $5a$.

Question 3:

Check whether $7 + 3x$ is a factor of $3x^3 + 7x$.

Solution 3:

Let us divide $(3x^3 + 7x)$ by $(7 + 3x)$.

By long division, we get

$$
\begin{array}{r|l}
 & x^2 - \frac{7}{3}x + \frac{70}{9} \\
\hline
3x+7 & 3x^3 + 0x^2 + 7x \\
 & 3x^3 + 7x^2 \\
 & - \quad - \\
\hline
 & -7x^2 + 7x \\
 & -7x^2 - \frac{49x}{3} \\
 & + \quad + \\
\hline
 & \frac{70x}{3} \\
 & \frac{70x}{3} + \frac{490}{9} \\
 & - \quad - \\
\hline
 & -\frac{490}{9} \\
\hline
\end{array}
$$

The remainder is not zero.

Therefore, $7 + 3x$ is not a factor of $3x^3 + 7x$.

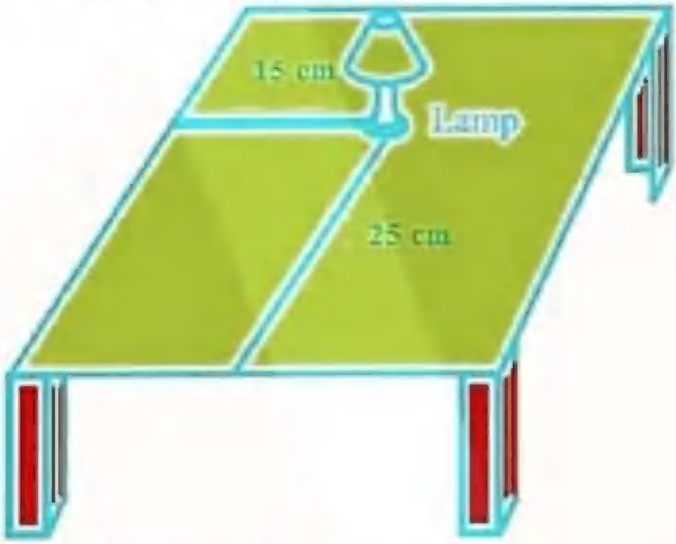
Exercise (3.1)

Question 1:

How will you describe the position of a table lamp on your study table to another person?

Solution 1:

Let us consider the given below figure of a study stable, on which a study lamp is placed.



From the Figure Above,

- Consider the lamp on the table as a point
- Consider the table as a plane.
- We can conclude that the table is rectangular in shape, when observed from the top.
    - The table has a short edge and a long edge.
    - Let us measure the distance of the lamp from the shorter edge and the longer edge.
    - Let us assume
        - Distance of the lamp from the shorter edge is 15 cm
        - Distance of the lamp from the longer edge, its 25 cm.

Therefore, we can conclude that the position of the lamp on the table can be described in two ways depending on the order of the axes as (15, 25) or (25, 15).

Question 2:

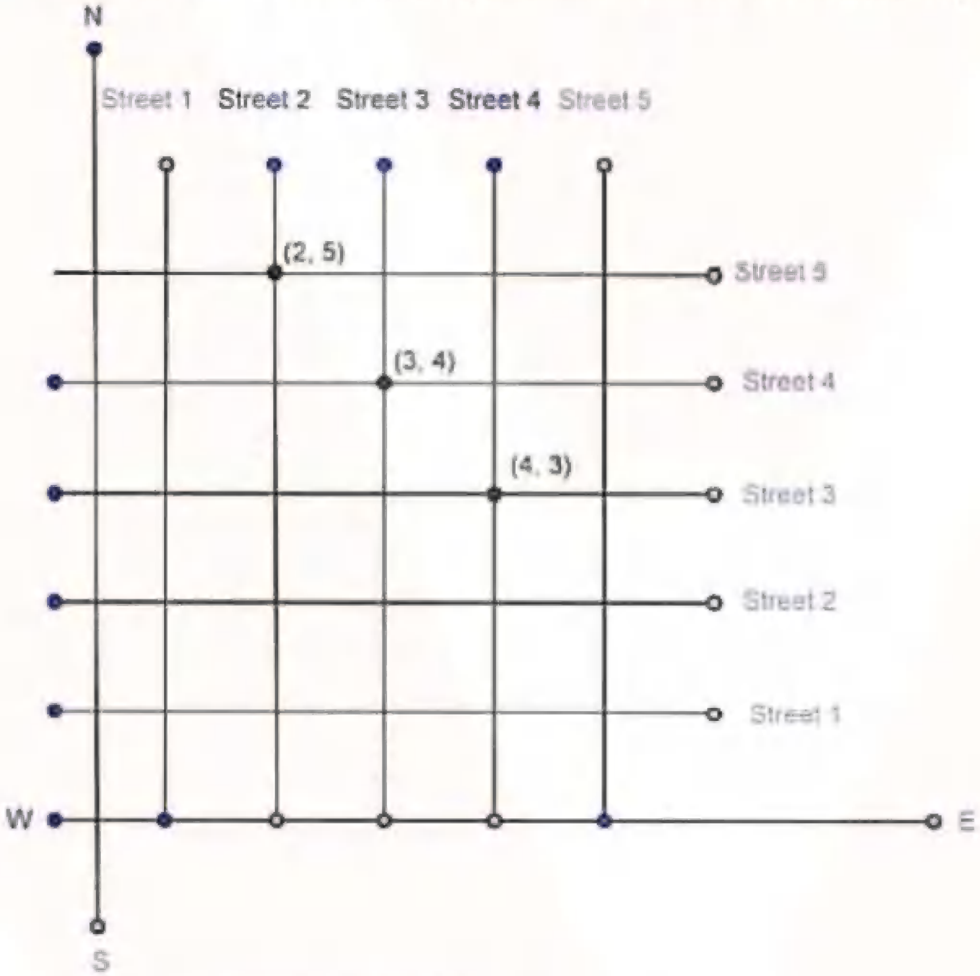
(Street Plan): A city has two main roads which cross each other at the center of the city. These two roads are along the North–South direction and East–West direction.

All the other streets of the city run parallel to these roads and are 200 m apart. There are 5 streets in each direction. Using 1 cm = 200 m, draw a model of the city on your notebook. Represent the roads/streets by single lines. There are many cross- streets in your model. A particular cross-street is made by two streets, one running in the North–South direction and another in the East–West direction. Each cross street is referred to in the following manner: If the 2nd street running

in the North–South direction and 5th in the East–West direction meet at some crossing, then we will call this cross-street (2, 5). Using this convention, find:
(i) How many cross - streets can be referred to as (4, 3).
(ii) How many cross - streets can be referred to as (3, 4).

Solution 2:

- Draw two perpendicular lines as the two main roads of the city that cross each other at the center
- Mark it as N–S and E–W.
- Let us take the scale as 1 cm = 200 m.
- Draw five streets that are parallel to both the main roads, to get the given below figure.



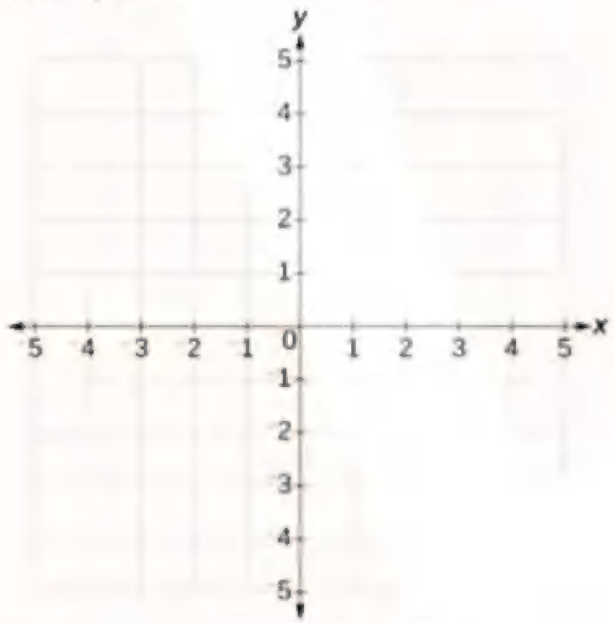
Street plan is as shown in the figure:
(i) There is only one cross street, which can be referred as (4, 3).
(ii) There is only one cross street, which can be referred as (3, 4).

Exercise (3.2)

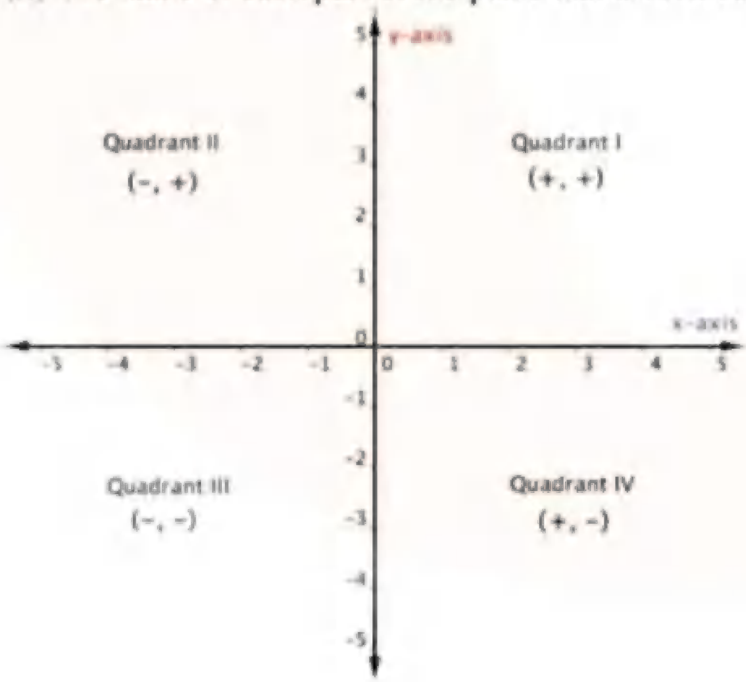
Question 1: Write the answer of each of the following questions:
(i) What is the name of horizontal and the vertical lines drawn to determine the position of any point in the Cartesian plane?
(ii) What is the name of each part of the plane formed by these two lines?
(iii) Write the name of the point where these two lines intersect.

Solution 1:
(i) The horizontal line that is drawn to determine the position of any point in the Cartesian plane is called as ***x*-axis**.

The vertical line that is drawn to determine the position of any point in the Cartesian plane is called as ***y*-axis**.

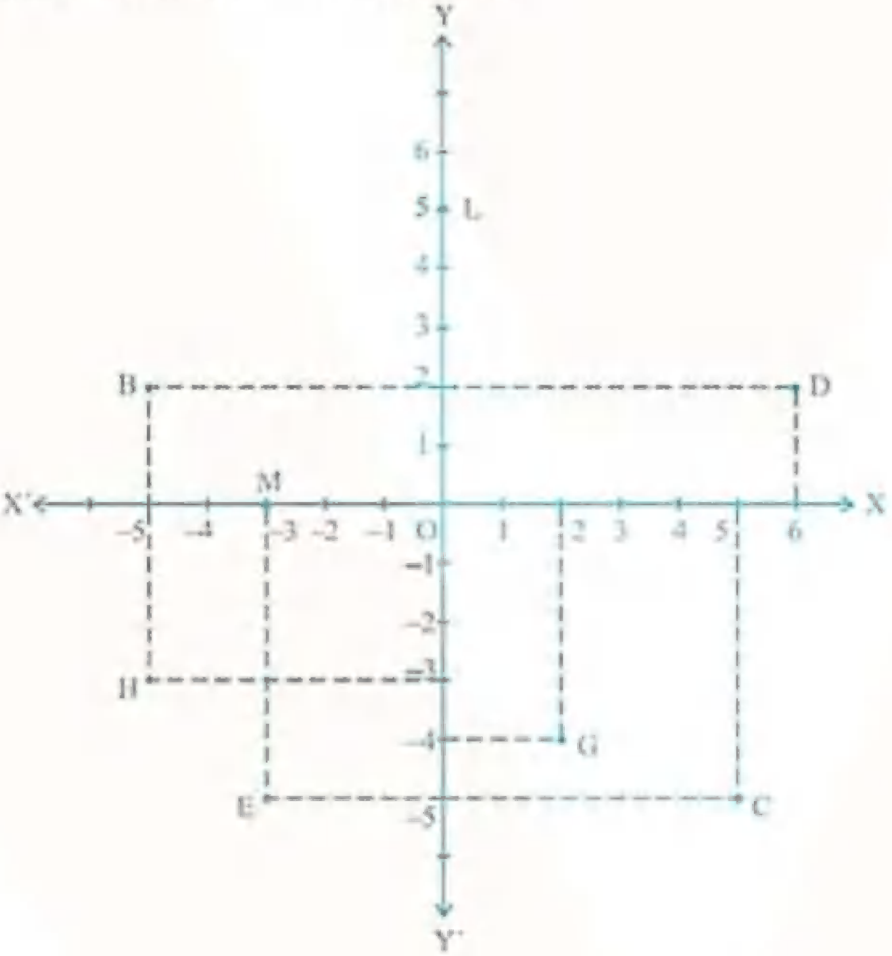


(ii) The name of each part of the plane that is formed by x-axis and y-axis is called as **quadrant**.



(iii) The point, where the x-axis and the y-axis intersect is called as **origin** (O)

Question 2: See the figure, and write the following:
(i) The coordinates of B.
(ii) The coordinates of C.
(iii) The point identified by the coordinates (–3, –5).
(iv) The point identified by the coordinates (2, –4).
(v) The abscissa of the point D.
(vi) The ordinate of the point H.
(vii) The coordinates of the point L.
(viii) The coordinates of the point M.



Solution 2:
From the Figure above,

(i) The coordinates of point B is the distance of point B from x-axis and y-axis. Therefore, the coordinates of point B are (–5, 2).
(ii) The coordinates of point C is the distance of point C from x-axis and y-axis. Therefore, the coordinates of point C are (5, –5).
(iii) The point that represents the coordinates (–3, –5) is E.
(iv) The point that represents the coordinates (2, –4) is G.

(v) The abscissa of point D is the distance of point D from the y-axis. Therefore, the abscissa of point D is 6.

(vi) The ordinate of point H is the distance of point H from the x-axis. Therefore, the abscissa of point H is –3.

(vii) The coordinates of point L in the above figure is the distance of point L from x-axis and y-axis. Therefore, the coordinates of point L are (0, 5).

(viii) The coordinates of point M in the above figure is the distance of point M from x-axis and y-axis. Therefore the coordinates of point M are (–3, 0).

Exercise (3.3)

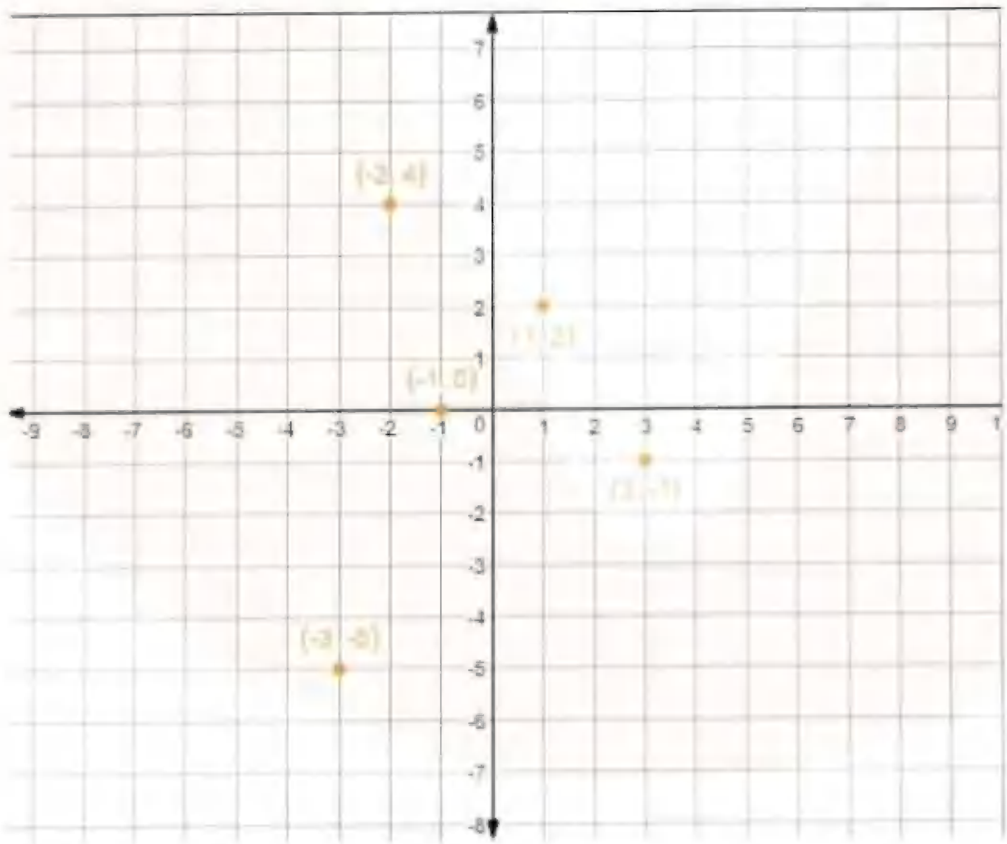
Question 1:

In which quadrant or on which axis do each of the points (–2, 4), (3, –1), (–1, 0), (1, 2) and (–3, –5) lie? Verify your answer by locating them on the Cartesian plane.

Solution 1:

To determine the quadrant or axis of the points (–2, 4), (3, –1), (–1, 0), (1, 2) and (–3, –5).

Plot the plot the points (–2, 4), (3, –1), (–1, 0), (1, 2) and (–3, –5) on the graph, to get



From the figure above, we can conclude that the points

- Point (–2, 4) lie in IInd quadrant.
- Point (3, –1) lie in IVth quadrant.
- Point (–1, 0) lie on the negative x-axis.
- Point (1, 2) lie in Ist quadrant.
- Point (–3, –5) lie in IIIrd quadrant.

Question 2:

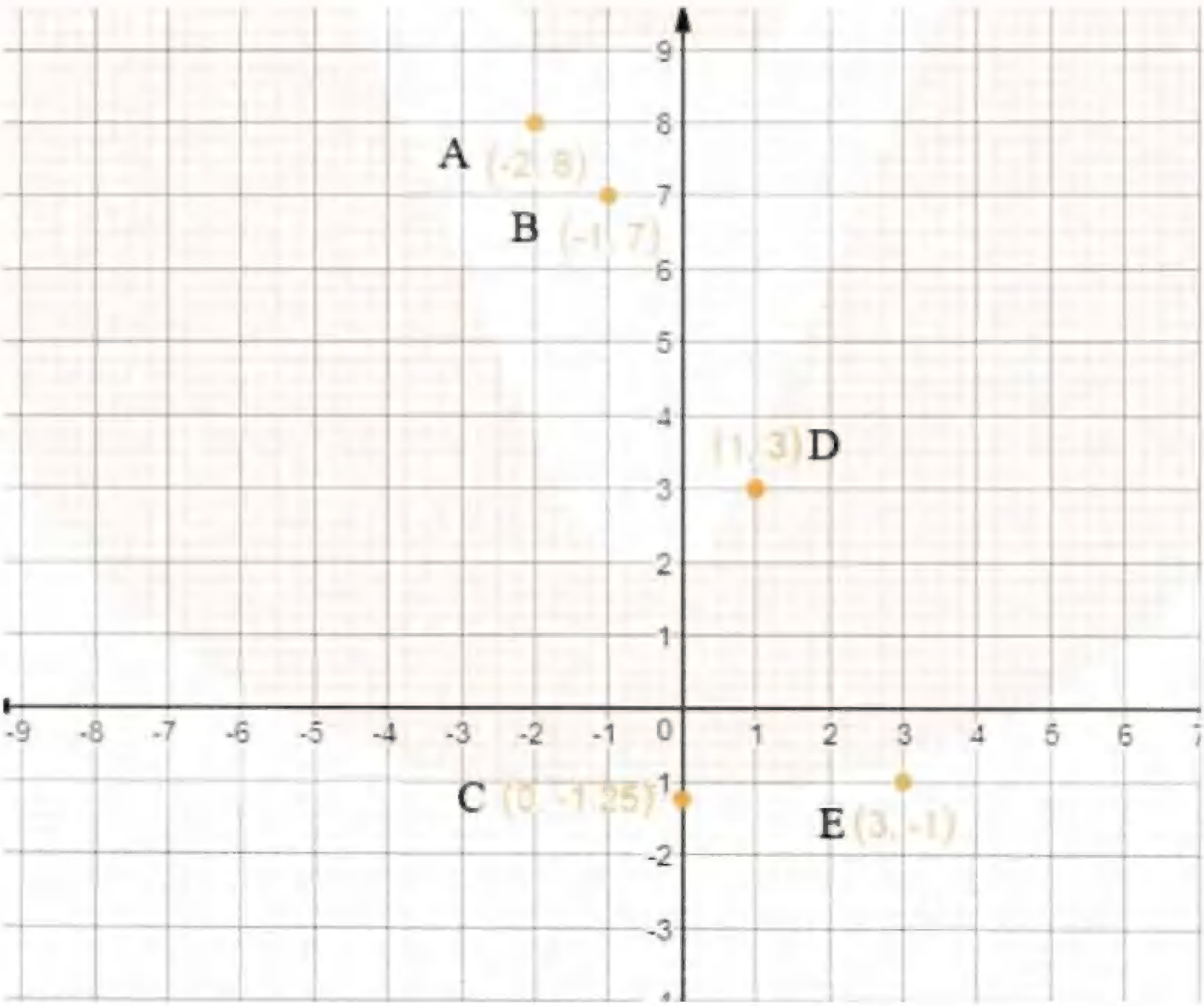
Plot the points (x, y) given in the following table on the plane, choosing suitable units of distance on the axes.

X	−2	−1	0	1	3
y	8	7	−1.25	3	−1

Solution 2:

Given,

X	−2	−1	0	1	3
y	8	7	−1.25	3	−1



Draw X'OX and Y'OY as the coordinate axes and mark their point of intersection O as the Origin (0, 0)

In order to plot the table provided above,

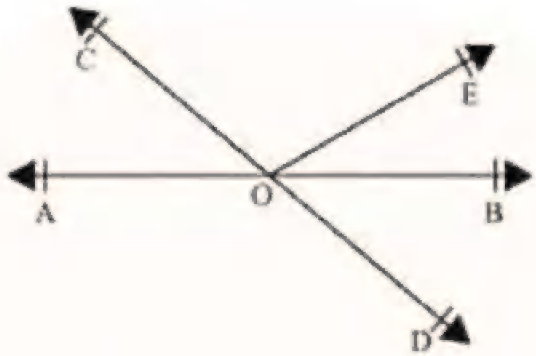
- Point A(−2, 8)
  - Take 2 units on OX' and then 8 units parallel to OY
- Point B (−1, 7)

  - Take -1 unit on OX' and then 7 units parallel to OY
- Point C(0, −1.25),
  - Take 1.25 units below x-axis on OY' on the y-axis
- Point D(1, 3)
  - Take 1 unit on OX and then 3 units parallel to OY
- Point E(3, −1),
  - Take 3 units on OX and then move 1 unit parallel to OY'

Exercise (6.1)

Question 1:

In the given figure, lines AB and CD intersect at O. If $\angle AOC + \angle BOE = 70^\circ$ and $\angle BOD = 40^\circ$ find $\angle BOE$ and reflex $\angle COE$.



Solution 1:

AB is a straight line, rays OC and OE stand on it.

$\therefore \angle AOC + \angle COE + \angle BOE = 180^\circ$

$\Rightarrow (\angle AOC + \angle BOE) + \angle COE = 180^\circ$

$\Rightarrow 70^\circ + \angle COE = 180^\circ$

$\Rightarrow \angle COE = 180^\circ - 70^\circ = 110^\circ$

Reflex $\angle COE = 360^\circ - 110^\circ = 250^\circ$

CD is a straight line, rays OE and OB stand on it.

$\therefore \angle COE + \angle BOE + \angle BOD = 180^\circ$

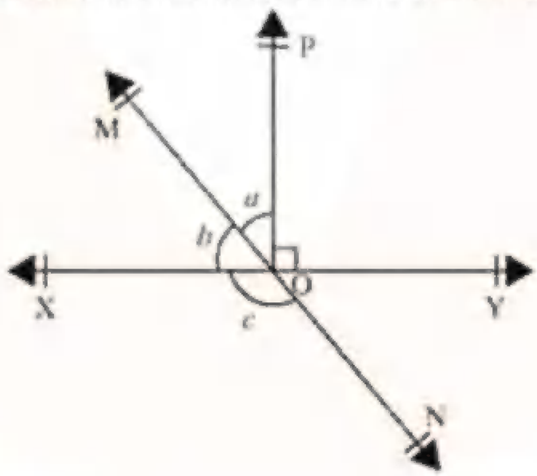
$\Rightarrow 110^\circ + \angle BOE + 40^\circ = 180^\circ$

$\Rightarrow \angle BOE = 180^\circ - 150^\circ = 30^\circ$

Hence, $\angle BOE = 30^\circ$ and Reflex $\angle COE = 250^\circ$

Question 2:

In the given figure, lines XY and MN intersect at O. If $\angle POY = 90^\circ$ and a: b = 2:3, find c.



Solution 2:

Let the common ratio between a and b be x.

$\therefore$ a = 2x, and b = 3x

XY is a straight line, rays OM and OP stand on it.

$\therefore \angle XOM + \angle MOP + \angle POY = 180^\circ$

$b + a + \angle POY = 180^\circ$

$3x + 2x + 90^\circ = 180^\circ$

$5x = 90^\circ$

$x = 18^\circ$

$a = 2x = 2 \times 18 = 36^\circ$

$b = 3x = 3 \times 18 = 54^\circ$

MN is a straight line. Ray OX stands on it.

$\therefore b + c = 180^\circ$ (Linear Pair)

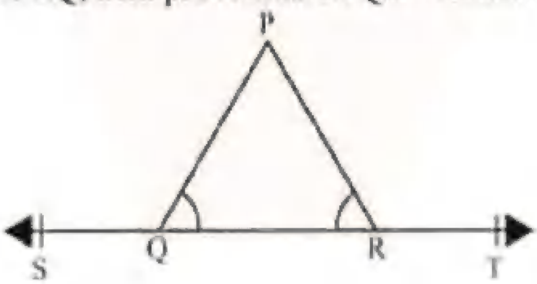
$54^\circ + c = 180^\circ$

$c = 180^\circ - 54^\circ = 126^\circ$

$\therefore c = 126^\circ$

Question 3:

In the given figure, $\angle PQR = \angle PRQ$, then prove that $\angle PQS = \angle PRT$.



Solution 3:

In the given figure, ST is a straight line and ray QP stands on it.

$\therefore \angle PQS + \angle PQR = 180^\circ$ (Linear Pair)

$\angle PQR = 180^\circ - \angle PQS$ ... (1)

$\angle PRT + \angle PRQ = 180^\circ$ (Linear Pair)

$\angle PRQ = 180^\circ - \angle PRT$ ... (2)

It is given that $\angle PQR = \angle PRQ$.

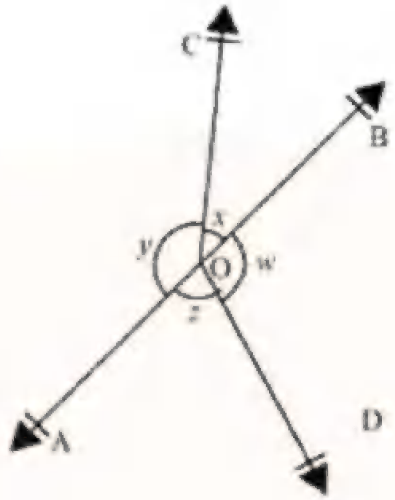
Equating Equations (1) and (2), we obtain

$180^\circ - \angle PQS = 180 - \angle PRT$

$\angle PQS = \angle PRT$ (Proved)

Question 4:

In the given figure, if $x + y = w + z$ then prove that AOB is a line.



Solution 4:

It can be observed that,

$x + y + z + w = 360°$ (Complete angle)

It is given that,

$x + y = z + w$

$\therefore\ x + y + x + y = 360°$

$2(x + y) = 360°$

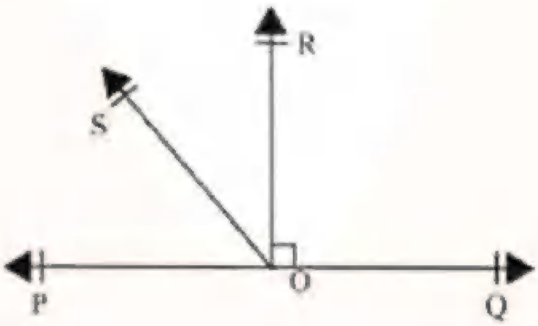
$x + y = 180°$

Since x and y form a linear pair, therefore, AOB is a line. (Proved)

Question 5:

In the given figure, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that

$\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$



Solution 5:

It is given that $OR \perp PQ$

$\angle POR = 90°$

$\angle POS + \angle SOR = 90°$

$\angle ROS = 90° - \angle POS$ ... (1)

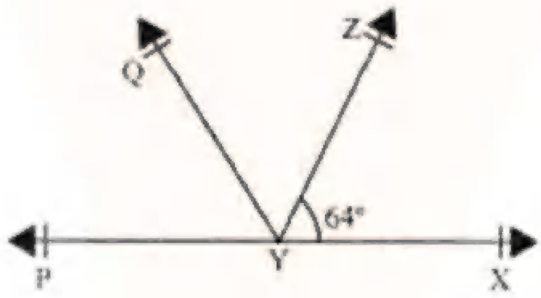
$\angle QOR = 90°$ (As $OR \perp PQ$)

$\angle QOS - \angle ROS = 90°$
$\angle ROS = \angle QOS - 90°$ ... (2)
On adding Equations (1) and (2), we obtain
$2\angle ROS = \angle QOS - \angle POS$
$$\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$$

Question 6:

It is given that $\angle XYZ = 64°$ and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects $\angle ZYP$, find $\angle XYQ$ and reflex $\angle QYP$.

Solution 6:

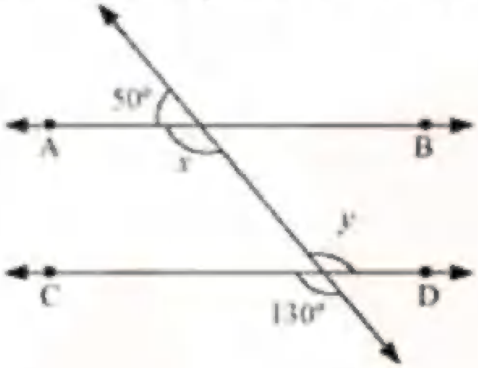


It is given that line YQ bisects $\angle PYZ$.
Hence, $\angle QYP = \angle ZYQ$
It can be observed that PX is a line. Rays YQ and YZ stand on it.
$\angle XYZ + \angle ZYQ + \angle QYP = 180°$
$64° + 2\angle QYP = 180°$
$2\angle QYP = 180° - 64° = 116°$
$\angle QYP = 58°$
Also, $\angle ZYQ = \angle QYP = 58°$
Reflex $\angle QYP = 360° - 58° = 302°$
$\angle XYQ = \angle XYZ + \angle ZYQ$
$= 64° + 58° = 122°$
Hence, $\angle XYQ = 122°$, Reflex $\angle QYP = 302°$

Exercise (6.2)

Question 1:

In the given figure, find the values of x and y and then show that AB || CD.



Solution 1:

It can be observed that,

$50° + x = 180°$ (Linear pair)

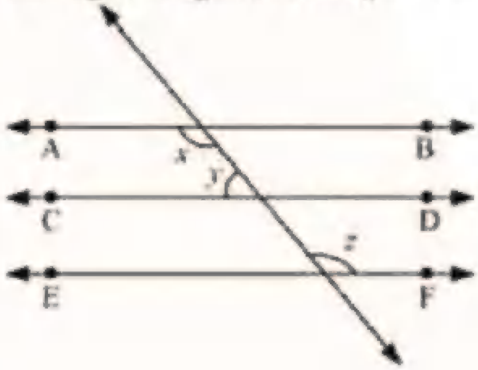
$x = 130°$ ... (1)

Also, $y = 130°$ (Vertically opposite angles)

As x and y are alternate interior angles for lines AB and CD and also measures of these angles are equal to each other, therefore, line AB || CD.

Question 2:

In the given figure, if AB || CD, CD || EF and $y : z = 3:7$, find x.



Solution 2:

It is given that AB || CD and CD || EF

∴ AB || CD || EF (Lines parallel to the same line are parallel to each other)

It can be observed that

$x = z$ (Alternate interior angles) ... (1)

It is given that $y : z = 3 : 7$

Let the common ratio between y and z be a.

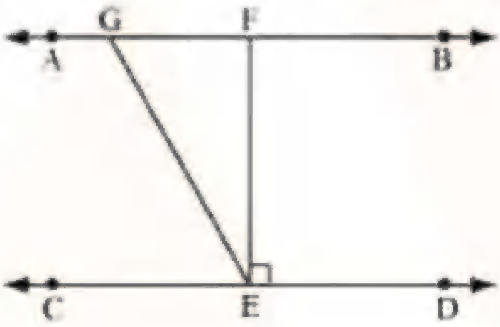
∴ $y = 3a$ and $z = 7a$

Also, $x + y = 180°$ (Co-interior angles on the same side of the transversal)

$z + y = 180°$ [Using Equation (1)]

Question 3:

In the given figure, If AB || CD, EF $\perp$ CD and $\angle GED = 126°$, find $\angle AGE$, $\angle GEF$ and $\angle FGE$.



Solution 3:

It is given that,

AB || CD and EF $\perp$ CD

$\angle GED = 126°$

$\angle GEF + \angle FED = 126°$

$\angle GEF + 90° = 126°$

$\angle GEF = 36°$

As $\angle AGE$ and $\angle GED$ are alternate interior angles.

$\angle AGE = \angle GED = 126°$

However, $\angle AGE + \angle FGE = 180°$ (Linear pair)

$126° + \angle FGE = 180°$

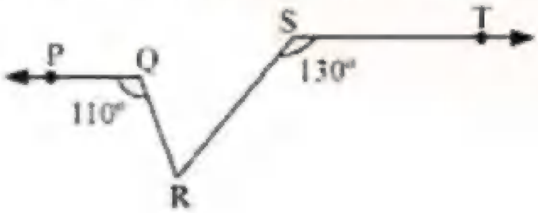
$\angle FGE = 180° - 126° = 54°$

Hence, $\angle AGE = 126°$, $\angle GEF = 36°$, $\angle FGE = 54°$

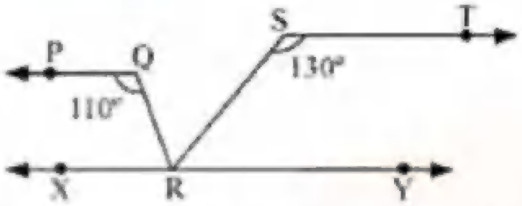
Question 4:

In the given figure, if PQ || ST, $\angle PQR = 110°$ and $\angle RST = 130°$, find $\angle QRS$.

[*Hint*: Draw a line parallel to ST through point R.]



Solution 4:



Let us draw a line XY parallel to ST and passing through point R.

$\angle PQR + \angle QRX = 180^\circ$ (Co-interior angles on the same side of transversal QR)

$110^\circ + \angle QRX = 180^\circ$

$\angle QRX = 70^\circ$

Also,

$\angle RST + \angle SRY = 180^\circ$ (Co-interior angles on the same side of transversal SR)

$130^\circ + \angle SRY = 180^\circ$

$\angle SRY = 50^\circ$

XY is a straight line. RQ and RS stand on it.

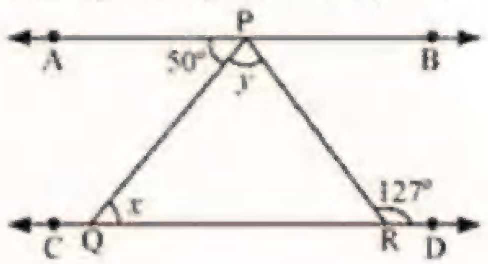
$\angle QRX + \angle QRS + \angle SRY = 180^\circ$

$70^\circ + \angle QRS + 50^\circ = 180^\circ$

$\angle QRS = 180^\circ - 120^\circ = 60^\circ$

Question 5:

In the given figure, if AB || CD, $\angle APQ = 50^\circ$ and $\angle PRD = 127^\circ$, find x and y.



Solution 5:

$\angle APR = \angle PRD$ (Alternate interior angles)

$50^\circ + y = 127^\circ$

$y = 127^\circ - 50^\circ$

$y = 77^\circ$

Also,

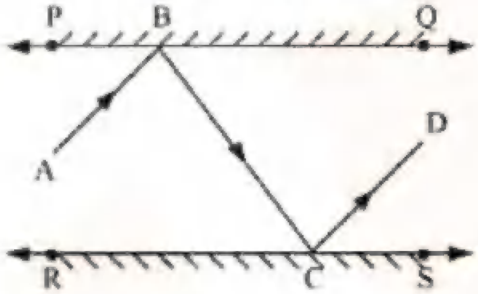
$\angle APQ = \angle PQR$ (Alternate interior angles)

$50^\circ = x$

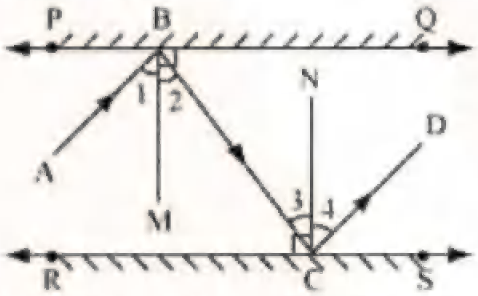
Therefore, $x = 50^\circ$ and $y = 77^\circ$

Question 6:

In the given figure, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that AB || CD.



Solution 6:



Let us draw BM $\perp$ PQ and CN $\perp$ RS.

As PQ || RS,

Therefore, BM || CN

Thus, BM and CN are two parallel lines and a transversal line BC cuts them at B and C respectively.

$\angle 2 = \angle 3$ (Alternate interior angles)

However, $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$ (By laws of reflection)

$\angle 1 = \angle 2 = \angle 3 = \angle 4$

Also,

$\angle 1 + \angle 2 = \angle 3 + \angle 4$

$\angle ABC = \angle DCB$

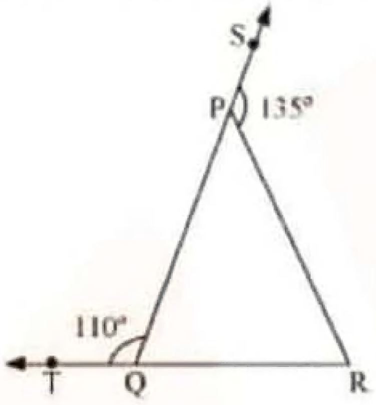
However, these are alternate interior angles.

$\therefore$ AB || CD

Exercise (6.3)

Question 1:

In the given figure, sides QP and RQ of ΔPQR are produced to points S and T respectively. If $\angle SPR = 135^\circ$ and $\angle PQT = 110^\circ$, find $\angle PRQ$.



Solution 1:

It is given that,

$\angle SPR = 135^\circ$ and $\angle PQT = 110^\circ$

$\angle SPR + \angle QPR = 180^\circ$ (Linear pair angles)

$135^\circ + \angle QPR = 180^\circ$

$\angle QPR = 45^\circ$

Also, $\angle PQT + \angle PQR = 180^\circ$ (Linear pair angles)

$110^\circ + \angle PQR = 180^\circ$

$\angle PQR = 70^\circ$

As the sum of all interior angles of a triangle is 180°, therefore, for ΔPQR,

$\angle QPR + \angle PQR + \angle PRQ = 180^\circ$

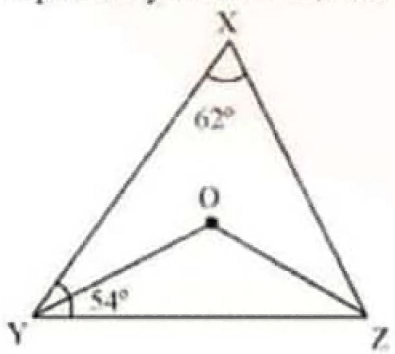
$45^\circ + 70^\circ + \angle PRQ = 180^\circ$

$\angle PRQ = 180^\circ - 115^\circ$

$\angle PRQ = 65^\circ$

Question 2:

In the given figure, $\angle X = 62^\circ$, $\angle XYZ = 54^\circ$. If YO and ZO are the bisectors of $\angle XYZ$ and $\angle XZY$ respectively of ΔXYZ, find $\angle OZY$ and $\angle YOZ$.



Solution 2:

As the sum of all interior angles of a triangle is 180°, therefore, for ΔXYZ,

$\angle X + \angle XYZ + \angle XZY = 180^\circ$

$62^\circ + 54^\circ + \angle XZY = 180^\circ$

$\angle XZY = 180^\circ - 116^\circ$

$\angle XZY = 64^\circ$

$\angle OZY = \dfrac{64}{2} = 32^\circ$ (OZ is the angle bisector of $\angle XZY$)

Similarly, $\angle OYZ = \dfrac{54}{2} = 27^\circ$

Using angle sum property for ΔOYZ, we obtain

$\angle OYZ + \angle YOZ + \angle OZY = 180^\circ$

$27^\circ + \angle YOZ + 32^\circ = 180^\circ$

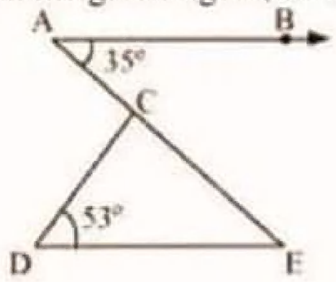
$\angle YOZ = 180^\circ - 59^\circ$

$\angle YOZ = 121^\circ$

Hence, $\angle OZY = 32^\circ$ and $\angle YOZ = 121^\circ$

Question 3:

In the given figure, if AB || DE, $\angle BAC = 35^\circ$ and $\angle CDE = 53^\circ$, find $\angle DCE$.



Solution 3:

AB || DE and AE is a transversal.

$\angle BAC = \angle CED$ (Alternate interior angles)

$\angle CED = 35^\circ$

In ΔCDE,

$\angle CDE + \angle CED + \angle DCE = 180^\circ$ (Angle sum property of a triangle)

$53^\circ + 35^\circ + \angle DCE = 180^\circ$

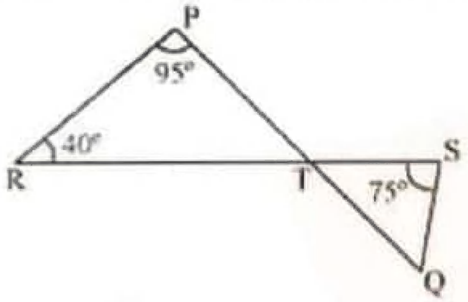
$\angle DCE = 180^\circ - 88^\circ$

$\angle DCE = 92^\circ$

Question 4:

In the given figure, if lines PQ and RS intersect at point T, such that $\angle PRT = 40^\circ$,

$\angle RPT = 95^\circ$ and $\angle TSQ = 75^\circ$, find $\angle SQT$.



Solution 4:

Using angle sum property for ΔPRT, we obtain

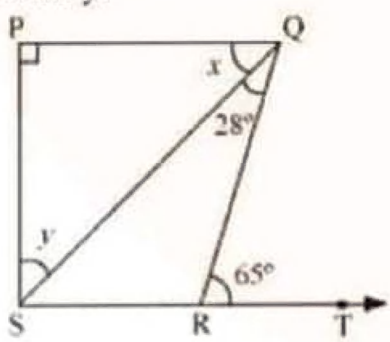
$\angle PRT + \angle RPT + \angle PTR = 180^\circ$
$40^\circ + 95^\circ + \angle PTR = 180^\circ$
$\angle PTR = 180^\circ - 135^\circ$
$\angle PTR = 45^\circ$
$\angle STQ = \angle PTR = 45^\circ$ (Vertically opposite angles)
$\angle STQ = 45^\circ$

By using angle sum property for ΔSTQ, we obtain

$\angle STQ + \angle SQT + \angle QST = 180^\circ$
$45^\circ + \angle SQT + 75^\circ = 180^\circ$
$\angle SQT = 180^\circ - 120^\circ$
$\angle SQT = 60^\circ$

Question 5:

In the given figure, if $PQ \perp PS$, $PQ \parallel SR$, $\angle SQR = 28^\circ$ and $\angle QRT = 65^\circ$, then find the values of x and y.



Solution 5:

It is given that $PQ \parallel SR$ and QR is a transversal line.
$\angle PQR = \angle QRT$ (Alternate interior angles)
$x + 28^\circ = 65^\circ$

$x = 65^{\circ} - 28^{\circ}$

$x = 37^{\circ}$

By using the angle sum property for ΔSPQ, we obtain

$\angle SPQ + x + y = 180^{\circ}$

$90^{\circ} + 37^{\circ} + y = 180^{\circ}$

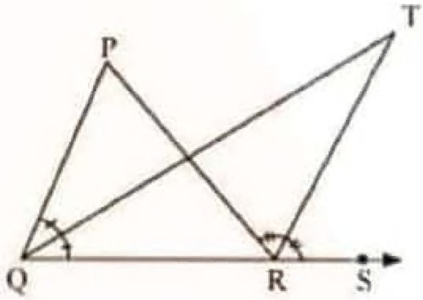
$y = 180^{\circ} - 127^{\circ}$

$y = 53^{\circ}$

$x = 37^{\circ}$ and $y = 53^{\circ}$

Question 6:

In the given figure, the side QR of ΔPQR is produced to a point S. If the bisectors of $\angle PQR$ and $\angle PRS$ meet at point T, then prove that $\angle QTR = \frac{1}{2}\angle QPR$.



Solution 6:

In ΔQTR, $\angle TRS$ is an exterior angle.

$\therefore \angle QTR + \angle TQR = \angle TRS$

$\angle QTR = \angle TRS - \angle TQR \quad \ldots (1)$

For ΔPQR, $\angle PRS$ is an external angle.

$\therefore \angle QPR + \angle PQR = \angle PRS$

$\angle QPR + 2\angle TQR = 2\angle TRS$ (As QT and RT are angle bisectors)

$\angle QPR = 2(\angle TRS - \angle TQR)$

$\angle QPR = 2\angle QTR$ [By using Equation (1)]

$\angle QTR = \frac{1}{2}\angle QPR$